5.2 Scaling Up the Compost Box

Goals

- Understand the effect on surface area of applying a scale factor to a rectangular prism
- Understand the effect on volume of applying a scale factor to a rectangular prism
- Extend students' understanding of similarity to three-dimensional figures

In this problem, students generalize the findings from Problem 5.1 and look at what happens when the dimensions of a 1-2-3 box are multiplied by various scale factors. If the same number multiplies each dimension, the resulting box is similar to a 1-2-3 box.

A Pro-Worm Testimonial

A teacher from Oregon wrote the following: "Ever since I heard about worm boxes, I had planned on starting one. This activity offered a great excuse. I had a book on composting that suggested a ratio of 2.5 pounds of paper per cubic foot of space and 3 pounds of water for every pound of paper. My box was 20 inches by 24 inches by 12 inches, so I started doing my math to figure out what I had to do to set this thing up. What a great activity for the kids. I went into class with a scale, newspaper, soil, two gallons of water, a mixing bin, some garbage from my kitchen and, of course, the worm box and worms. I was wearing a pin that said, 'Worms eat my garbage.' Kids noticed. They were excited. My pet worms are now happily digesting garbage under my table at school."

Launch 5.2

Suggested Questions Hold up the 1-2-3 and 2-4-6 boxes. Ask:

- *Are these two boxes mathematically similar?* (various answers)
- *How can we check?* (Measure and compare corresponding sides. Look for a common scale factor.)
- *What is the relationship between the corresponding dimensions of the two boxes?* (The larger box's dimensions are twice those of the smaller box.)
- *What is the relationship between the surface areas of the two boxes?* (The surface area of the larger box is 4 times that of the smaller box.)
- *What is the relationship between the volumes of the two boxes?* (The volume of the larger box is 8 times the volume of the smaller box.)
- *What is the scale factor from the 1-2-3 to the 2-4-6 box?* (The scale factor is 2.)
- *So, are these two boxes similar?* (These two boxes are similar because each dimension is multiplied by the same factor: 2.)

Demonstrate the volume relationship by stacking eight 1-2-3 boxes inside the 2-4-6 box.

Tell the class that Ms. Fernandez's class is building a variety of similar compost boxes. Remind students that the compost boxes are open on top.

- *What is the surface area of a 1-2-3 box?* (16 square feet)
- *What is the volume of a 1-2-3 box?* (6 cubic feet)
- *What feature of the box does the amount of garbage that is decomposed each day relate to?* (volume)
- *If we double the amount of garbage, what feature of the 1-2-3 box will we need to double?* (volume)
- *If we double the dimensions of the 1-2-3 box, will we also double the garbage it can decompose?* (No, we will increase the amount of garbage it can decompose—a function of the volume—by a factor of 8, not 2.)

Ask the class what they think will happen to the volume and surface area of a 1-2-3 box if its dimensions are tripled or quadrupled.

- *If you tripled each dimension of a 1-2-3 box, what would happen to the surface area?* (It would be 9 times the original, or 3^2.)
- *How do you know?*

- *If you tripled each dimension of a 1-2-3 box, what would happen to the volume?* (It would be 27 times the original, or 3^3.)
- *How do you know?*
- *How much plywood would be needed to make the new box?* (144 ft^2)
- *How many pounds of garbage could the new box decompose?* (27 times 0.5, or 13.5 lb; the amount of garbage is similar to the volume.)

Let the class offer a few conjectures. Tell them that these types of questions are what they will be exploring in this problem.

Let the class work in groups of 3 to 4.

Explore 5.2

The students in each group should work together to find the volumes and surface areas of all the boxes that the group designs. If students are struggling with this problem, you may need to review the properties of similar rectangles. The idea of similar rectangular prisms should follow naturally.

Summarize 5.2

Put up the chart.

Suggested Questions Help the class summarize what they have found. Ask:

- *What happens to the volume of a box if its dimensions are tripled?* (The volume will be 27 times as great. It would take 27 of the original boxes to fill the new box.)
- *What happens to the volume if the dimensions are quadrupled?* (The volume will be 64 times as great. It would take 64 of the original boxes to fill the new box.)
- *What happens to the surface area of a box if its dimensions are tripled?* (The surface area will be 9 times as great.)
- *What happens to the surface area if the dimensions are quadrupled?* (The surface area will be 16 times as great.)

Ask questions to assess how well students are grasping the ideas of similarity and scale factor as applied to three-dimensional figures.

- *What are the dimensions, surface area, and volume of a box similar to a 1-2-3 box if the scale factor is 9?* (The dimensions are 9 feet by 18 feet by 27 feet. The surface area is $16 \times 81 = 1{,}296$ square feet, and the volume is $6 \times 729 = 4{,}374$ cubic feet.)
- *What are the dimensions of a box that is similar to a 1-2-3 box but has a surface area of 1600 square feet? What is the scale factor from the 1-2-3 box to the new box?* (The surface area of a 1-2-3 box is 16 square feet, so the scale factor from the small box to the large box is 10. This means that the new box's dimensions are 10 feet by 20 feet by 30 feet.)
- *What are the dimensions of a box that is similar to a 1-2-3 box but has a volume of 10,368 cubic feet? What is the scale factor from the 1-2-3 box to the new box?* (The scale factor is $10{,}368 \div 6 = 1{,}728$. We need to find a number whose cube is 1,728. We know that 103 is 1,000, so it must be greater than 10 but not much. 113 is 1,331, and 123 is 1,728, so the scale factor is 12. The new box's dimensions are 12 feet by 24 feet by 36 feet.)
- *If the scale factor from a 1-2-3 box to a larger box is 4, how many 1-2-3 boxes will fit in the large box?* (The new box will hold $4 \times 4 \times 4 = 64$ of the 1-2-3 boxes.)
- *Which measurement is the change in garbage associated with?* (volume)

5.2 Scaling Up the Compost Box

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Extend students' understanding of similarity to three-dimensional figures
- Understand the effect on surface area of applying a scale factor to a rectangular prism
- Understand the effect on volume of applying a scale factor to a rectangular prism

Launch

Materials
- Transparency 5.2
- Labsheet 5.2
- 1-2-3 and 2-4-6 boxes
- Boxes from Problem 5.1

Hold up the 1-2-3 and 2-4-6 boxes.

- *Are these two boxes mathematically similar?*
- *How can we check?*
- *What is the relationship between the corresponding dimensions of the two boxes?*
- *What is the relationship between the surface areas of the two boxes?*
- *What is the relationship between the volumes of the two boxes?*
- *What is the scale factor from the 1-2-3 to the 2-4-6 box? So, are these two boxes similar?*

Demonstrate the volume relationship by stacking eight 1-2-3 boxes inside the 2-4-6 box. Tell the class that Ms. Fernandez's class is building a variety of similar compost boxes. Remind students that the compost boxes are open on top.

- *What feature of the box does the amount of garbage that is decomposed each day relate to?*
- *If we double the amount of garbage, what feature of the 1-2-3 box will we need to double?*

Ask the class what they think will happen to the volume and surface area of a 1-2-3 box if its dimensions are tripled or quadrupled.

Let the class offer a few conjectures. Tell them that these types of questions are what they will be exploring in this problem.

Have students work in groups of 3 or 4.

Explore

The students in each group should work together to find the volumes and surface areas of all the boxes that the group designs. If students are struggling with this problem, you may need to further discuss the properties of similar rectangles. The idea of similar rectangular prisms should follow naturally.

Summarize

Put up the chart. Help the class summarize what they have found.

- *What happens to the volume of a box if its dimensions are tripled?*
- *What happens to the volume if the dimensions are quadrupled?*
- *What happens to the surface area of a box if its dimensions are tripled?*
- *What happens to the surface area if the dimensions are quadrupled?*

Ask questions to assess how well students are grasping the ideas of similarity and scale factor as applied to three-dimensional figures.

Materials

- Student notebooks

ACE Assignment Guide for Problem 5.2

Differentiated Instruction
Solutions for All Learners

Core 8, 10–14
Other *Applications* 5–7, 9, 15–17; *Connections* 27; *Extensions* 36, 37; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 17 and other ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 5.2

A. (Figure 1)

B. Once the surface area of the 1-2-3 box is determined, the surface area of a similar box is *original surface area* $\times$ (*scale factor*)^2. For example, the surface area of the original 1-2-3 box is 16 ft^2. The surface area of the 2-4-6 box, which has been increased by a scale factor of 2, is 64 ft^2 or 16×2^2. The surface area of the 3-6-9 box, where 3 is the scale factor, is 144 ft^2, or 16×3^2. Having a closed compost box will not change the factor.

C. Once the volume of the original 1-2-3 box is determined, the volume of a similar box is *original volume* $\times$ (*scale factor*)^3. For example, the volume of the original 1-2-3 box is 6 ft^3. The volume of the 2-4-6 box, which has been increased by a scale factor of 2, is 48 ft^3 or 6×2^3. The volume of the 3-6-9 box, where the scale factor is 3, is 162 ft^3 or 6×3^3.

D. The amount of garbage that can be decomposed in a day is related to the volume of the box. Thus, once the amount of decomposed garbage for the original 1-2-3 box is determined, the amount for a similar box is the original amount $\times$ (scale factor)^3. The amount of decomposed garbage in the original 1-2-3 box is $\frac{1}{2}$ pound of garbage per day. The amount of garbage decomposed in the 2-4-6 box, which has been increased by a scale factor of 2, is 4 lbs. or $\frac{1}{2} \times 2^3$.

E. If the scale factor between the 1-2-3 box and a similar box is N, for the similar box, the new dimensions are: N-2N-3N, its surface area is N^2 times the surface area of the 1-2-3 box, and its volume is N^3 times the volume of the 1-2-3 box.

Figure 1

Compost Box Project

Open Box (*h-w-l*)	Scale Factor	Surface Area (sq. ft)	Volume (cu. ft)	Amount of Garbage Decomposed in a Day (lb)	Number of Worms Needed
1-2-3	1	16	6	0.5	1,000
2-4-6	2	64	48	4	8,000
3-6-9	3	144	162	13.5	27,000
4-8-12	4	256	384	32	64,000
8-16-24	8	1,024	3,072	256	512,000
10-20-30	10	1,600	6,000	500	1,000,000